SPINORS ON LOOP SPACE

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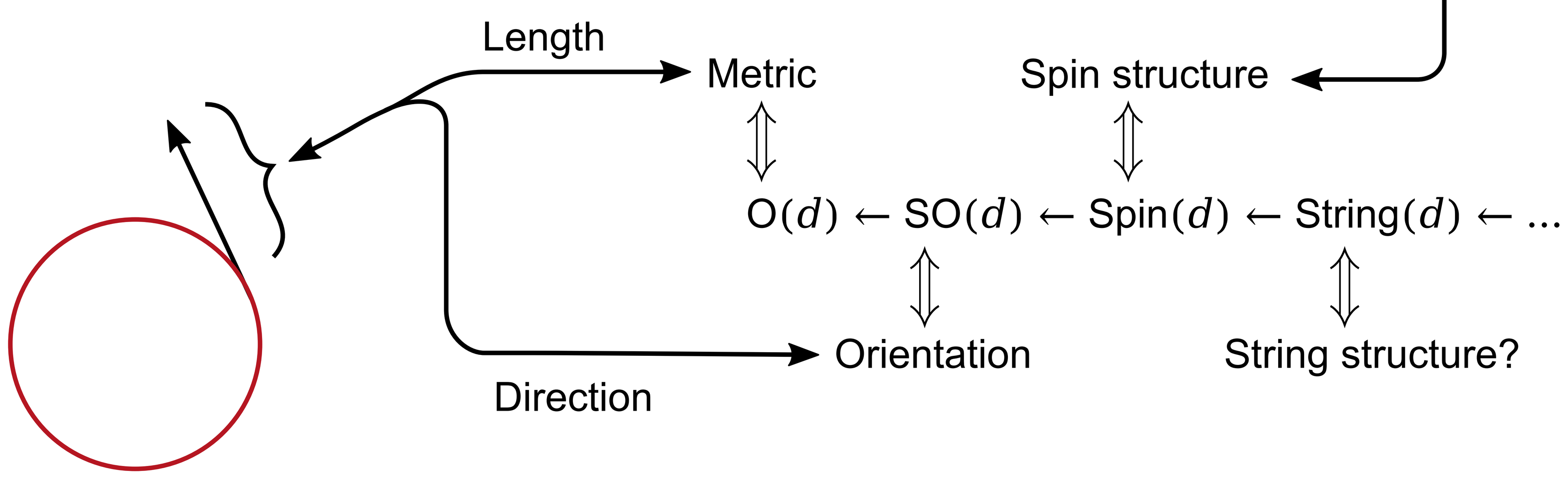
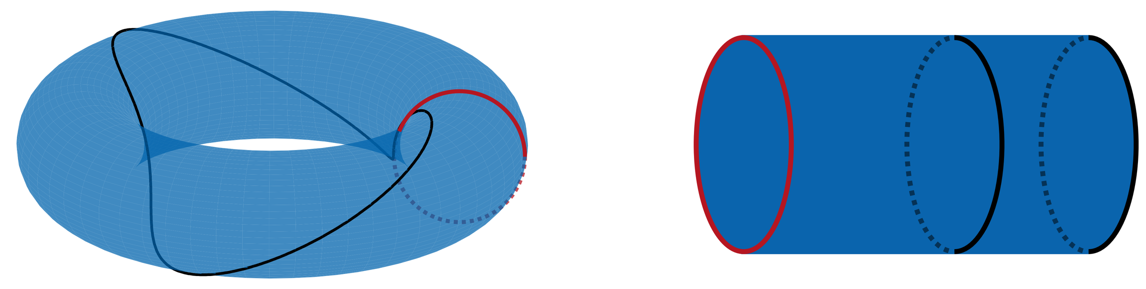
Motivation

Given a manifold, we try to equip it with more and more structure. In this case, we try to find reductions and lifts of the structure group of its tangent bundle along the Whitehead tower of $\mathrm{O}(d)$.

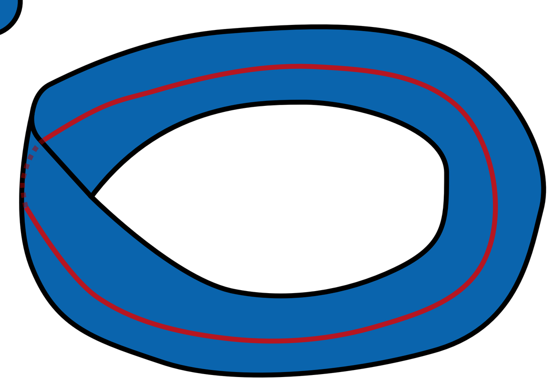




The odd (left) and even (right) spin structure on the circle in red.
To each point in the red circle belong two points in the black space.

Fermions are described by spinor bundles.

The Möbius strip is the spinor bundle associated to the odd spin structure. Write $\mathbb{S}$ for its complexification.



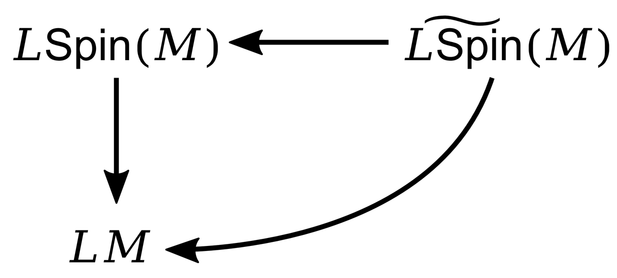
$\mathrm{String}(d)$ is complicated, we understand $\widetilde{L\mathrm{Spin}}(d)$ much better.

⇕

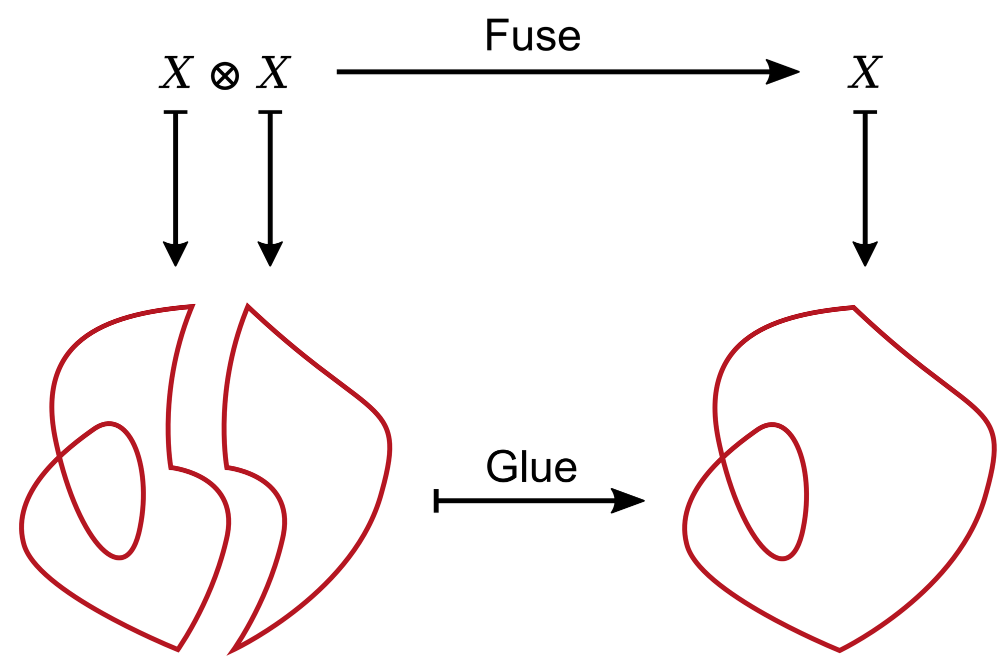
Transgression turns a string structure on a manifold into a fusion spin structure on its loop space.

Fusion Spin

A fusion spin structure on the loop space of a spin manifold is a lift of the looped spin frame bundle to an $\widetilde{L\mathrm{Spin}}(d)$ bundle, equipped with a fusion map.



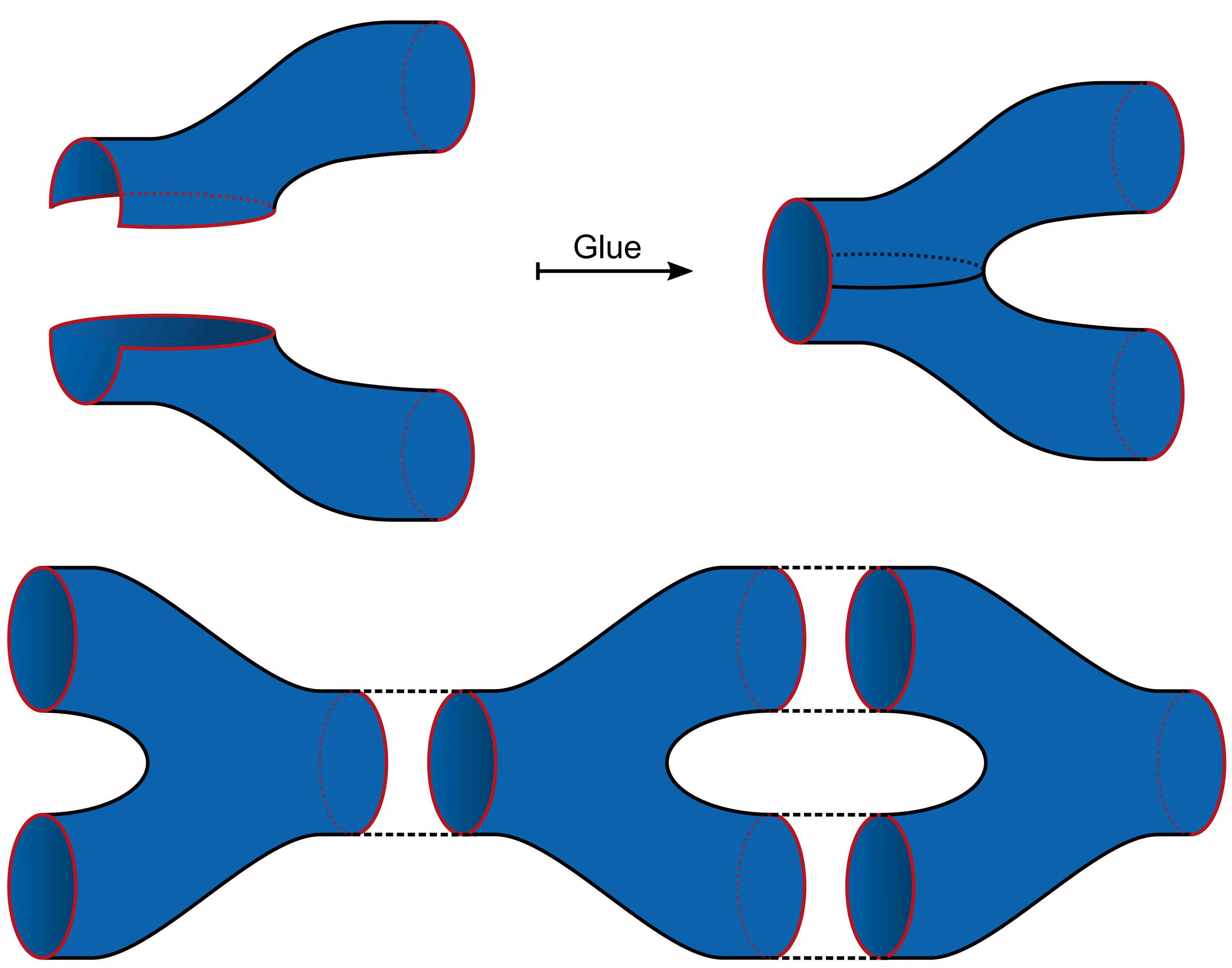
Fusion



If we have a fibre bundle over loop space, then fusion is a map that covers the gluing map.

Functorial Quantum Field Theory

To a loop $\gamma \in LM$ we assign a Hilbert space $\mathcal{F}_\gamma$, which should be seen as its space of states. To a compact oriented surface with boundary, we assign a linear operator. By gluing along circles and semicircles we can construct any compact oriented surface from disks and cylinders. This means it is sufficient to define linear operators for cylinders and disks.



$\gamma \longrightarrow$
$\mathcal{H}_\gamma = L^2(S^1, \mathbb{S} \otimes \gamma^* TM_{\mathbb{C}})$
$D_\gamma = i\partial_t \otimes \gamma^* \nabla_{\mathrm{LC}}$
$L_\gamma = \mathrm{Eig}_{>0}(D_\gamma)$
$\mathcal{F}_\gamma = \Lambda L_\gamma$

$\Sigma \longrightarrow$
$L_\Sigma = \mathrm{Hol}(D^2, \mathbb{D} \otimes \Sigma^* TM_{\mathbb{C}})$
$\Lambda L_\gamma \cong \Lambda L_\Sigma$
$\Omega = 1 \in \Lambda^0 L_\Sigma \subset \Lambda L_\Sigma$

Start with a section over γ_{in}.
Extend it to a holomorphic section over the surface.
Restrict to a section over γ_{out}.